Abstract

We present the Natural Deduction Assistant (NaDeA) and discuss its advantages and disadvantages as a tool for teaching logic. In particular we compare our approach to natural deduction in the Isabelle proof assistant. NaDeA is available online:

https://nadea.compute.dtu.dk/

1 Introduction

The Natural Deduction Assistant (NaDeA) [5, 6] runs in a standard browser and is open source software (https://github.com/logic-tools/nadea). Our formalization in the Isabelle proof assistant [4] of the syntax, semantics and the inductive definition of the natural deduction proof system extends work by Berghofer [1] but with a much more detailed soundness proof that can be examined and tested by the students. The corresponding completeness proof is also available but it is of course quite demanding. NaDeA can be used with or without installing Isabelle and it is not necessary that the students have any knowledge about proof assistants [2].

2 A Sample Proof

We consider the following formula and its online proof here: https://nadea.compute.dtu.dk/

∃x.A(x) → (∀x.A(x))

Click Cancel help, Load, Test 9 and Load shown proof in order to obtain the finished proof.

Figure 1 shows the start of the proof – more or less - where a natural deduction rule is to be chosen in the proof step 2.

Figure 2 shows the finished proof – one can always click on Undo to go back all the way to the very first proof step 1.

![Natural Deduction Assistant](https://nadea.compute.dtu.dk/)

Figure 1: A Sample Proof – Start
3 Selected Features for Students

We briefly describe a number of NaDeA features for students:

- Figure 3 shows the Welcome window. The Help button brings up the help window with this welcome information and a number of so-called hints.

- Figure 4 shows the Tutorial window. It contains a getting started guide as well as a list of the natural deduction primitives.

- Figure 5 shows the Exercises window. Solutions to all exercises are provided and can be revealed step-by-step with hints.

- Figure 6 shows the special NaDeA, soundness and completeness, window. The so-called verification button allows the user to verify any finished proof in Isabelle.

- Figure 7 shows the major Isabelle Code window – entitled: Definition of natural deduction proof system – with the formalization in Isabelle.

- Figure 8 shows the minor Isabelle Code window – entitled: Definition of first-order logic syntax and semantics – with the formalization in Isabelle.

There are several other NaDeA features for students – for example, the ProofJudge system in NaDeA can manage student assignments in courses with teaching assistants.

NaDeA uses the automation of our verified declarative prover [3] tool to give students feedback on the provability of their goals and subgoals. Overall both that prover and NaDeA are related to the IsaFoL (https://bitbucket.org/isafol) project which unites researchers in formalizing logic in Isabelle. Among the formalizations in the project are SAT-solving, first-order resolution, a paraconsistent logic, sequent calculi and more.

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4 Conclusion

NaDeA has been used for teaching first-order logic to hundreds of computer science bachelor students. NaDeA has recently been used by a class of mainly PhD students at the 29th European Summer School in Logic, Language, and Information (ESSLLI), University of Toulouse, France, 17-28 July 2017 (https://www.irit.fr/esslli2017/courses/24.html).

Proof assistants such as Isabelle allow for many kinds of reasoning that go beyond natural deduction and their interfaces, of course, account for that. In NaDeA, on the other hand, there are no distractions – all buttons and texts in NaDeA have to do with natural deduction. The structured environment provided by NaDeA, based on clicking buttons instead of textual input, makes it possible for students to focus on the proof development process. Since NaDeA only allows input of well-formed formulas and application of applicable rules, the student does not have to worry about neither syntax or well-formedness errors possible for instance in Isabelle. Conversely, experienced users may feel slightly inhibited by the system as textually inputting a formula is often faster than using the mouse. Furthermore for very simple proofs, students may be able to find them by clicking blindly and without understanding what they are doing, since only the applicable rules are shown. This is not a problem for larger proofs.

As future work we consider developing more teaching materials and making further evaluations of NaDeA as a tool for teaching logic.

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References

Figure 3: Welcome

Figure 4: Tutorial
A test resumes from the last proof state but a hint reverts from the first proof state (click Undo to show the proof states).

Exercise

1. \( \vdash A \rightarrow A \)
2. \( \vdash \neg \vdash \)
3. \( A \vdash (A \rightarrow B) \rightarrow B \)
4. \( A \vdash (A \rightarrow B) \rightarrow (A \rightarrow B) \)
5. \( (A \rightarrow B) \rightarrow C \vdash (A \rightarrow B) \rightarrow (C \land B) \)
6. \( (A \rightarrow B) \vdash (A \rightarrow B) \rightarrow (A \rightarrow C) \)
7. \( A \vdash (A \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow (B \rightarrow C)) \)
8. \( (A \rightarrow A) \rightarrow (A \rightarrow A) \)
9. \( (A \rightarrow A) \rightarrow (A \rightarrow A) \)

Exercises are available as Hint 0.9 in this Help window and sample proofs are available as Test 0.9 in the Load window.

Tabs in the browser are useful to switch between the load window, the code window, the help windows and the main proof window.

NaDeA starts with a hint if the hash mark is and the hint number is added to the address in the browser address line.

Figure 6: NaDeA, soundness and completeness
Figure 7: Definition of natural deduction proof system

Figure 8: Definition of first-order logic syntax and semantics